P: ISSN NO.: 2394-0344 RNI No.UPBIL/2016/67980 VOL-I* ISSUE-IX* December- 2016
E: ISSN NO.: 2455-0817 Remarking An Analisation

Some Properties of the F- Structure Manifold Satisfying $F^{3k}+F^{k}=0$

Abstract

In this paper we have studied various properties of the F structure manifold satisfying $F^{3k}+F^{k}=0$, where K is a positive integer. The metric F structure, f induced on each integral manifold of tangent bundles l* have also been discussed.

Keywords: Differentiable Manifold, Projection Operators, Tangent Bundles and Metric.

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Introduction

Let V_n be a C^∞ differentiable manifold and F be a C^∞ (1,1) tensor defined on V_n, s.t.

(1.1) $F^{3k}+F^{k}=0.$

We define the projection operators l and m on v_n by

(1.2) $l= -F^{2k}$, $m= I+F^{2k}$

From (1.1) and (1.2), we have

(1.3) $l+m=I$, $l^2=l$, $m^2=m$, $lm=ml=0$

$lF^k= F^kl= F^k$, $mF^k=F^km=0$

Theorem (1.1) If rank ((F))=n, then

(1.4) l=I, m=0

Proof from the fact

(1.5) Rank ((F))+ nullity ((F))= dim V_n=n

(1.6) Nulity ((F))=0 =) Ker ((F))= {0}

Thus FX=0 =) X=0

Then let $FX_1 = FX_2$

=) F (x_1-x_2)=0

=) $X_1=X_2$ or F is 1-1.

Moreover V_n being finite dimensional , F is onto also : F is invertible =) F^k is inversibl, operating F^{-k} on $F^kl= F^k$ and $mF^k=0$, we get (1.4)

Theorem (1.2) let m and F satisfy

(1.7) $m^2=m$, $mF^k=F^km=0$, $(m+F^k)(m-F^k)=I$, then F satisfying (1.1)

Proof : $(m+F^k)(m-F^k)=I$

$m^2-mF^k+F^km-F^{2k}=I$

$m-0+0-F^{2k}=I$

$mF^k-F^{3k}=F^k$

$0-F^{3k}=f^k$

Or $F^{3k}+F^k=0.$

Metric F Structure

If we define

(2.1) $'F(X,Y)=g(FX, Y)$ is skew symmetric

then

(2.2) $g(FX,Y) = -g(X,FY)$

Theorem (2.1) with the definitions in (2.1) and (2.2) we have

(2.3) $g(F^kX, F^kY) = (-1)^{k+1} [g(X,Y)-m(X,Y)]$

Where

(2.4) $'m(X,Y)=g(mX,Y)$

Proof using (1.2), (1.3), (2.2) and (2.4), we have

(2.5) $g(F^kX, F^kY)= (-1)^k g(X, F^{2k}Y)$

$=(-1)^k g(X, -ly)$

$=(-1)^{k+1}g(X,ly)$

$=(-1)^{k+1}g(X, (I-m)Y)$

$=(-1)^{k+1}[g(X, Y)-g(X, mY)]$

$=(-1)^{k+1}[g(X, Y)-g(mX,Y)]$

$=(-1)^{k+1}[g(X, Y)-{}'m(X, Y)]$

Theorem (2.2) [F, g] is not unique.

Proof (2.6), Let $\mu F'=F\mu$, $'g(X, Y)=g(\mu X, \mu Y)$

Then from (1.1), (1.2), (1.3) and (2.6)

(2.7) $\mu F'^{3k}= F^{3k}\mu= -F^k\mu=-\mu F'^k$

Or

(2.8) $F^{/3k}+F^{/k}=0$. Also

(2.9) $^{/}g(F^{/k}x, F^{/k}Y)=g(\mu F^{/k}X, \mu F^{/k}Y)$
$=g(F^{k}\mu X, F^{k}\mu Y)$
$=(-1)^{k}g(\mu X, F^{2k}\mu Y)$
$=(-1)^{k}g(\mu X, -l\mu Y)$
$=(-1)^{k+1}g(\mu X, l\mu Y)$
$=(-1)^{k+1}g(\mu X, (I-m)\mu Y)$
$=(-1)^{k+1}[g(\mu X, \mu Y)-g(\mu X, m\mu Y)]$
$=(-1)^{k+1}[\,^{/}g(X, Y)-\,^{/}m^{/}(X,Y)]$

Induced Structure f, Define

(3.1) $fX^{/}=FX^{/}$ for $X^{/}\in l^{*}$

Theorem (3.1), If f satisfy (3.1) and F (1.1)

Then $[f^{k}]$ is an almost complex structure.

Proof From (1.2), (1.3) and (3.10

(3.2) $f^{2k}\, lX^{/}=F^{2k}lX^{/}$
$= -l^{2}X^{/}$
$= -lX^{/}$

Thus $[f^{k}]$ acts as an almost complex structure on l^{*}.

Also

(3.3) $\mu l^{/}=-\mu F^{/2k}$
$= -F^{2k}\mu$
$=l\mu$

(3.4) $\mu m^{/}= \mu\,(I+F^{/2k})$
$= \mu+\mu F^{/2k}$
$=\mu+ F^{2k}\mu$
$= m\mu$.

Aim of Study

The ain of study is to develope the conditions under which recovery of (1.1) is possible.

Conclusion

If $m^{2}=m$, $mF^{k}=F^{k}m=0$, $(m+F^{k})(m-F^{k})=I$, then (1.1) is recovered.

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